POLITECNICO DI MILANO

Thesis proposals





Advanced statistical and numerical methods for the analysis of high dimensional functional data, spatial data and complex object data

Laura M. Sangalli

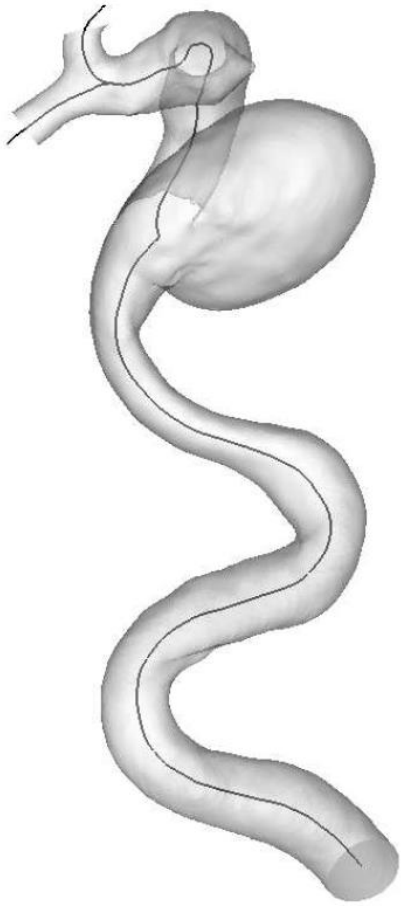
MODELLISTICA E CALCOLO SCIENTIFICO
MODELING AND SCIENTIFIC COMPUTING

Functional data: where they come from

Explosive growth in recording complex and high-dimensional data, e.g., having a functional nature (i.e., representable by curves, surfaces, dynamic curves and surfaces), non-euclidean data

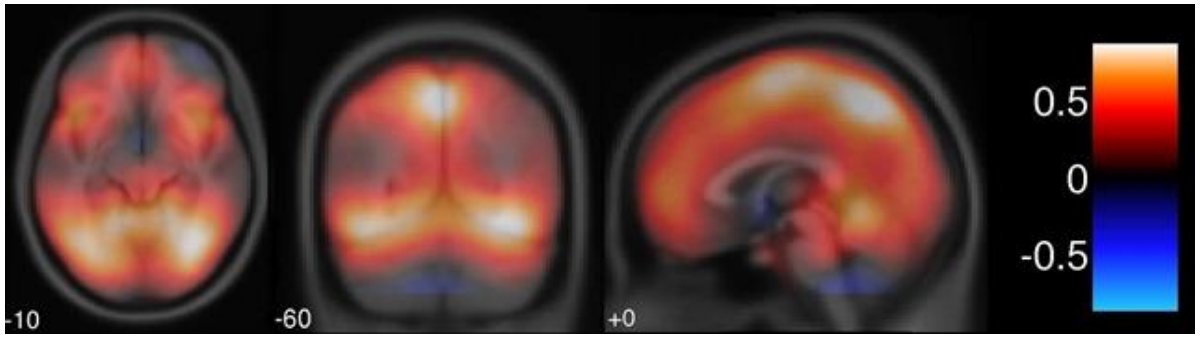
2D and 3D images and measures captured in time and space

► images of the internal structures of a body provided by diagnostic medical scanners



Reconstruction of an inner carotid artery with aneurysm, from angiographic images

Sangalli, Secchi, Vantini, Veneziani (2009) J. R. Stat. Soc. Ser. C

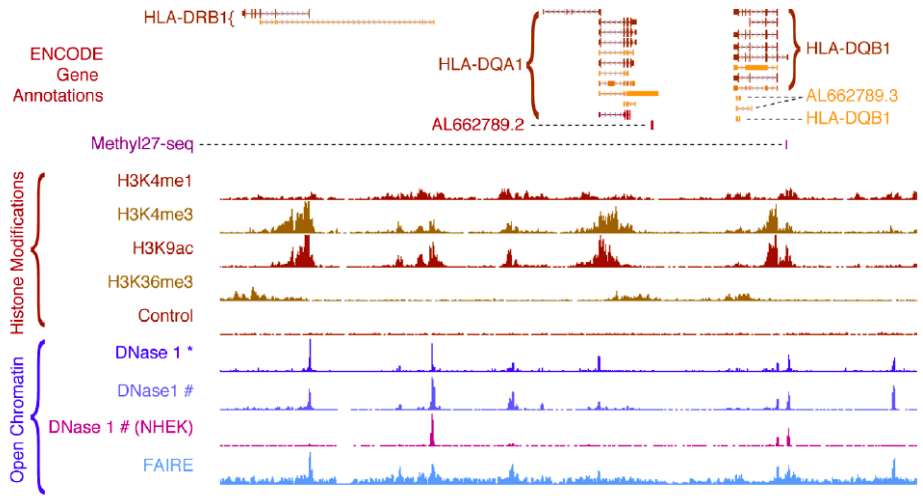


Magnetic Risonance Imaging of a brain during a reading task

Aston, Turkheimer, Brett (2006) Hum. Brain Map.

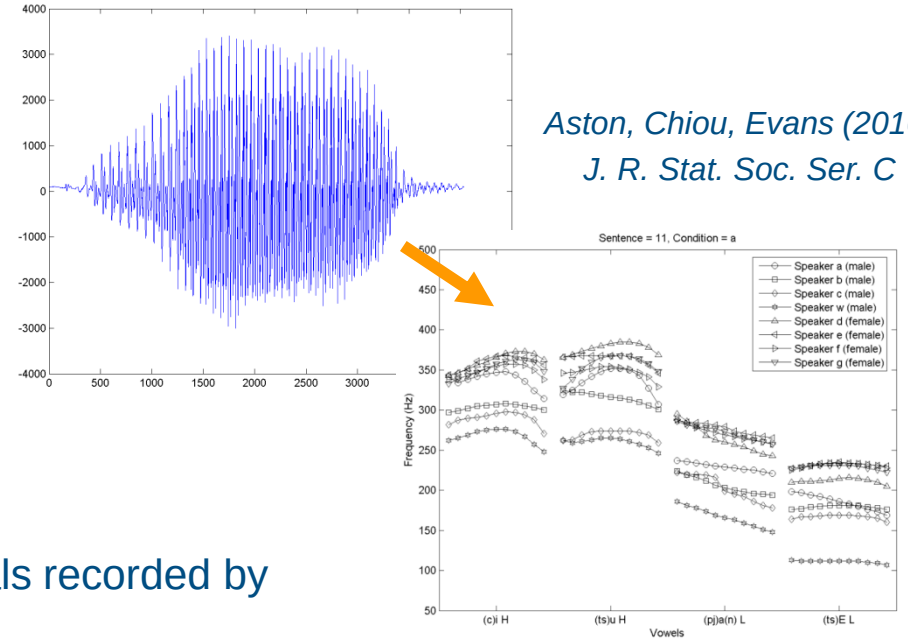
Functional data: where they come from

- measurements of gene expression levels



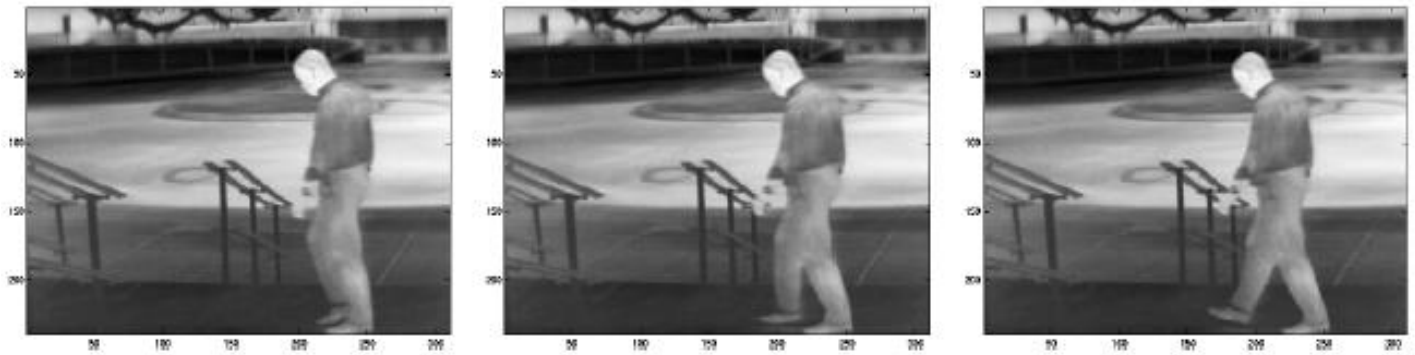
The ENCODE Project Consortium, 2011, PLoS Biology

- large speech databases describing linguistic constructs expressed via spectrum data



Aston, Chiou, Evans (2010)
J. R. Stat. Soc. Ser. C

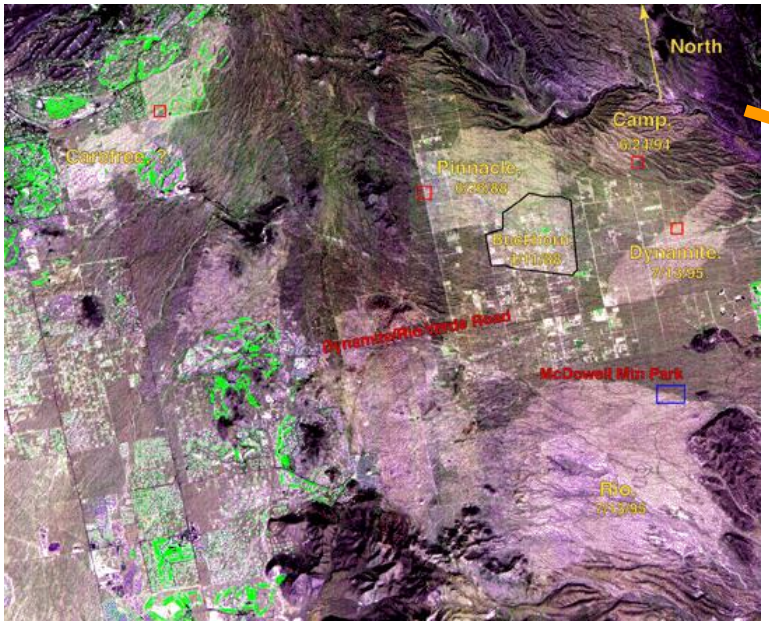
- images of steady or moving objects/individuals recorded by computer vision devices



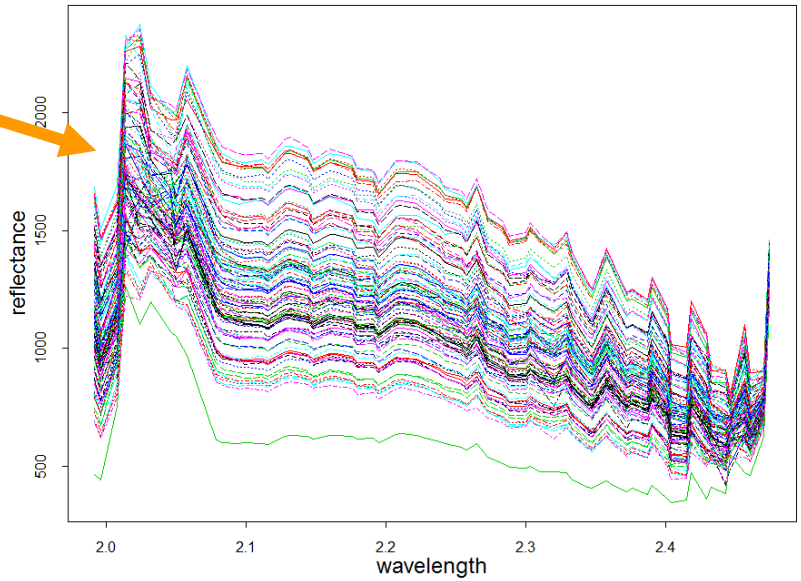
Kaziska, Srivastava (2007)
J. Amer. Statist. Assoc.

Functional data: where they come from

- multi-spectral data from satellite remote sensing



Northwest Scottsdale / Rio Verde area

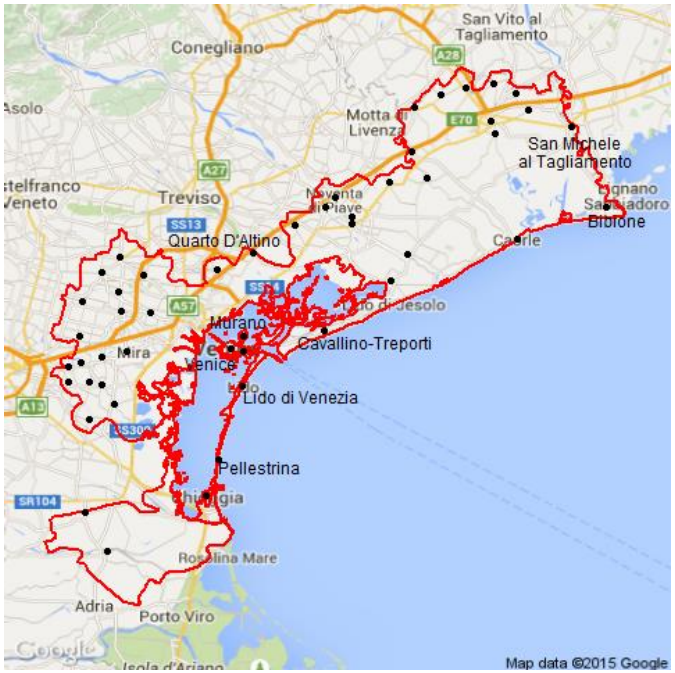


The analysis of complex and high dimensional data poses new and challenging problems in research

It is fueling one of the most fascinating and fast growing research fields of modern statistics

Spatial regression with differential regularization

- Modeling functional/spatial data with complex dependencies



- Domains with a complex structure
- Conditions at domain boundaries

→ Sangalli, Ramsay, Ramsay, 2013, JRSSB

→ Bernardi, Mazza, Ramsay, Sangalli, 2015, TechRep

Spatial regression with differential regularization

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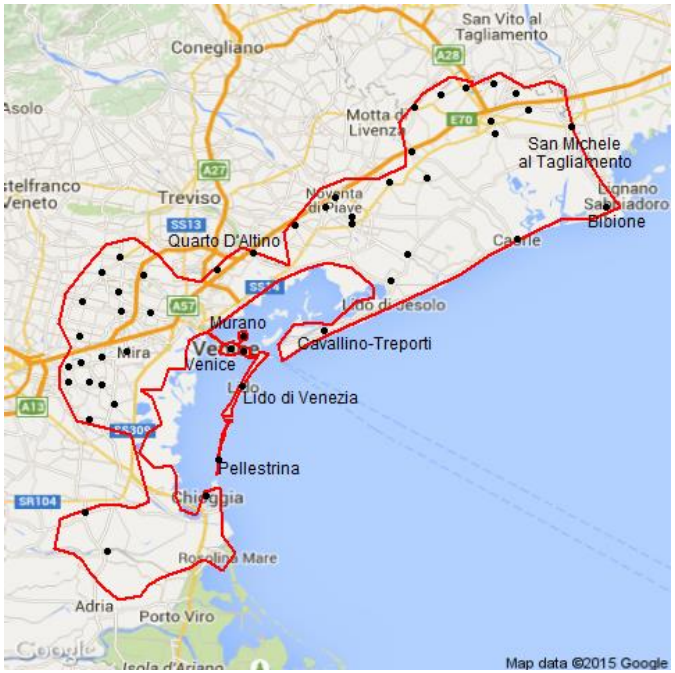

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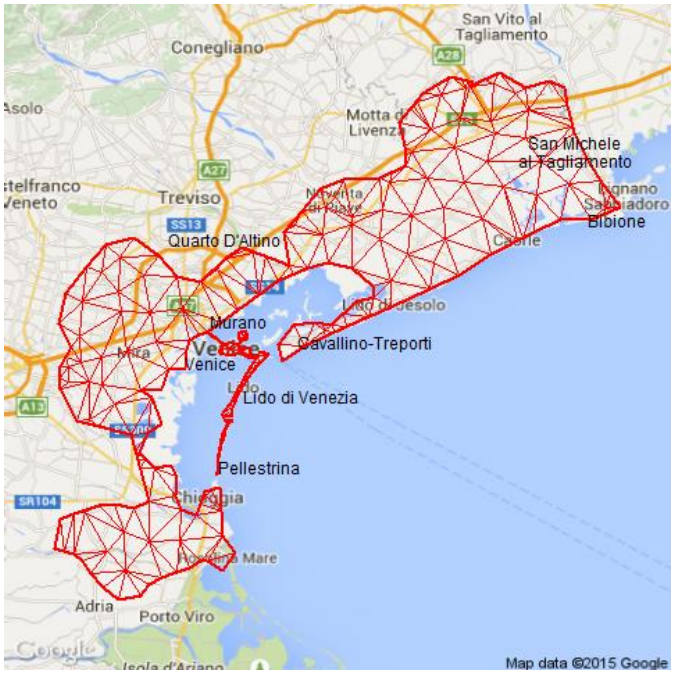

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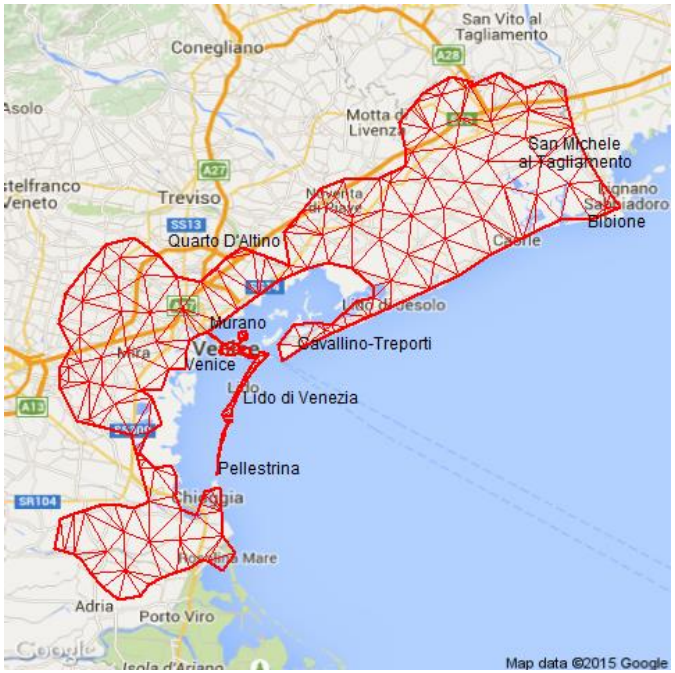

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- Riemannian manifolds domains

⟶ Ettinger, Perotto, Sangalli, 2015, Biometrika

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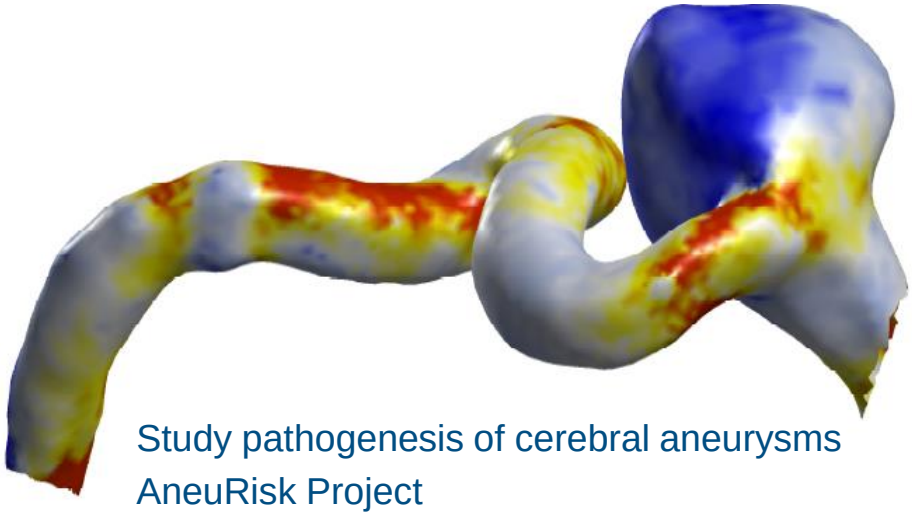

Study pathogenesis of cerebral aneurysms
AneuRisk Project

Spatial regression with differential regularization

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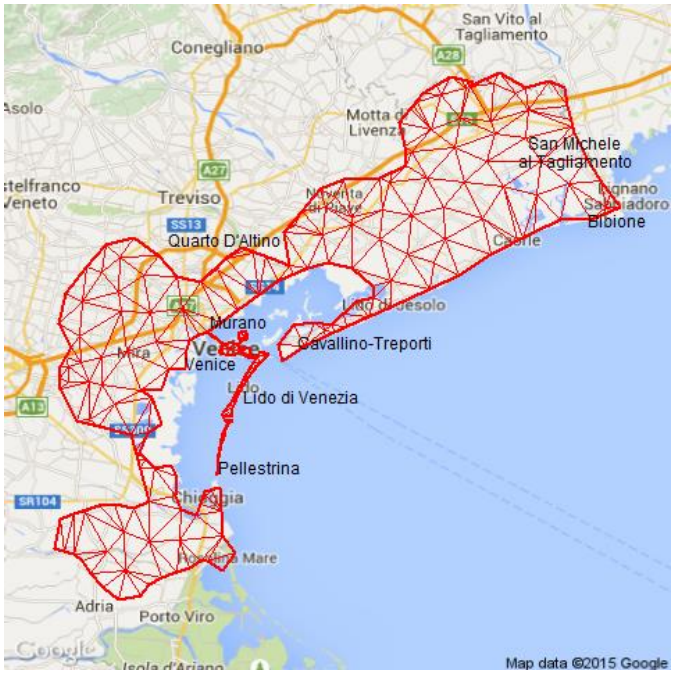

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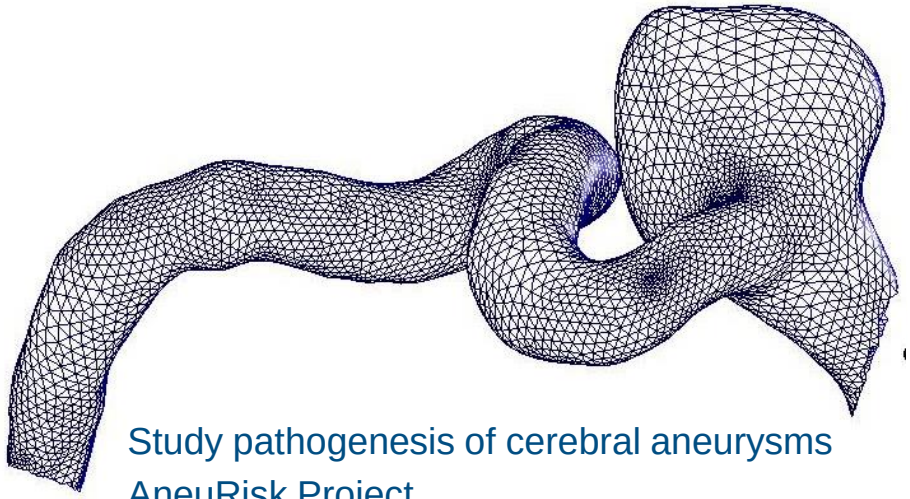

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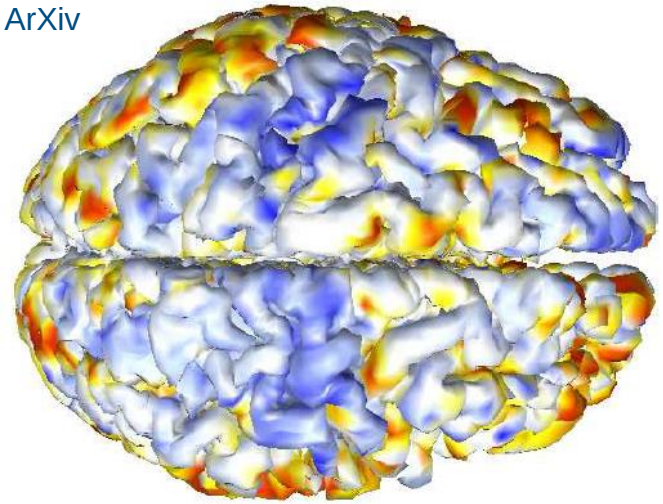
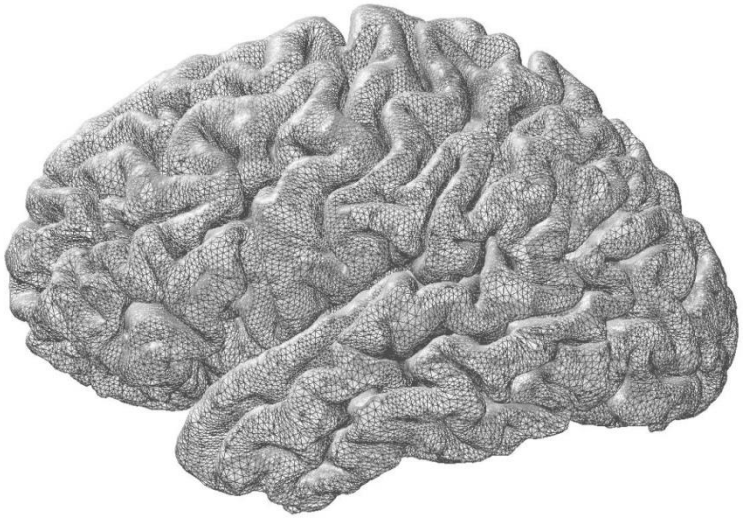
Spatial Regression models over Riemannian manifolds

- applications in the neurosciences

⟶ Dassi, Ettinger, Perotto, Sangalli, 2015, APNUM

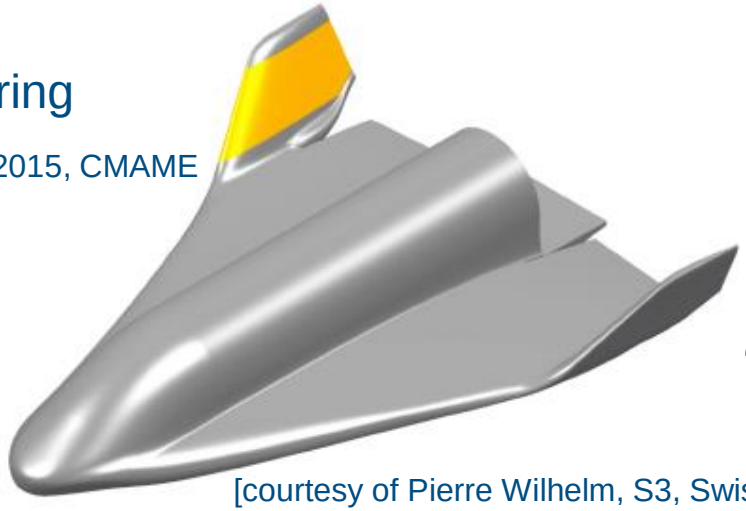
⟶ Lila, Aston, Sangalli, 2015, ArXiv

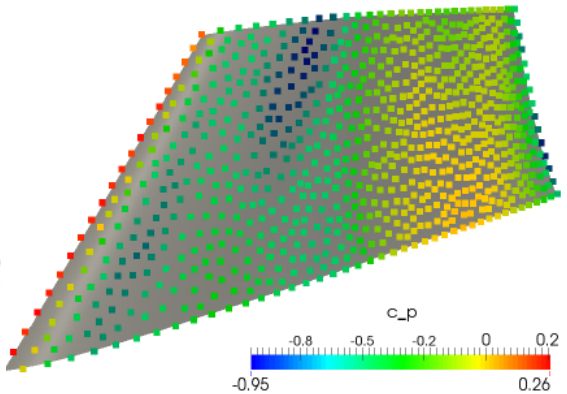




- applications in engineering

⟶ Wilhelm, Dedè, Sangalli, Wilhelm, 2015, CMAME





[courtesy of Pierre Wilhelm, S3, Swiss Space Systems Holding SA]

Spatial regression with differential regularization

- A piori knowledge about phenomenon under study allowing for very flexible modelling of space/time variation
  (anysotropy and non-stationarity)

  ⟶ Azzimonti, Sangalli, Secchi, Nobile, Domanin, 2014, JASA

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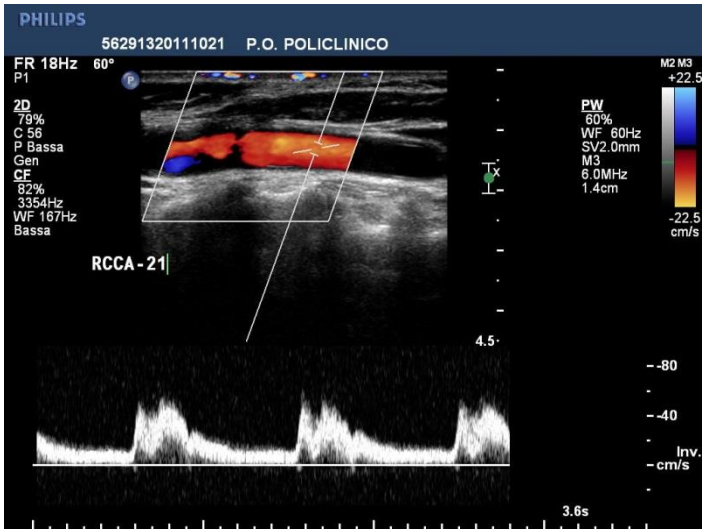

Study pathogenesis of atherosclerotic plaques

MAthematichs for CARotid ENdarterectomy @ MOX

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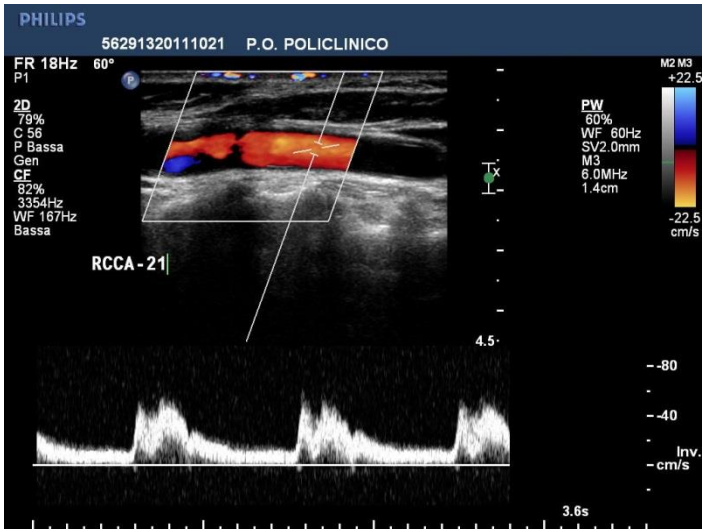

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- Space/time regression models with partial differential equation regularization
- Estimation problem solved resorting to Finite Elements or NURBS (iso-geometric analysis)
- Strong interplay of statistics with techniques from both pure math (analysis and differential geometry) and applied math (numerical analysis and scientific computing), and engineering

A FEW POSSIBLE TOPICS (many others can be considered, according to the candidate interests)

► Extension to volume data, using volumetric FE (tetrahedral, hexahedral elements, or more advanced hybrid elements that might be particularly convenient for specific fields of applications): possible applications e.g. to neurosciences, to the study of neuroimaging signal in the brain volume, accurately complying with the complex shape of this organ

► Extension to complex data objects (e.g., manifold valued data): possible applications in the neurosciences and earth-sciences

► Extension to multiple realizations of the signals over the complex domain, to allow for population study: possible applications in the neurosciences and other life-sciences

► Estimation of the iperparameters in PDE regularizing term (connections with the field of uncertainty quantification in complex models governed by PDEs)

► Derivation of explicit formulation for space covariance structure implied by differential regularization ()

► Study of regularity conditions ensuring that the differential regularization provides a valid covariance model

► Study of asymptotic properties

[Available code: R package (with C++ code), Matlab]

- Thesis can be
  - mostly or solely applied work
  - mostly implementative work
  - mostly methodological/theoretical work
  - any desired mix of the above
- Thesis can be
  - mostly on the statistical side
  - mostly on the numerical analysis side (with co-supervision by a colleague in Numerical Analysis)
  - any desired mix of the above
- Thesis can be combined with the course projects of
  - Applied Statistics
  - Advanced Programming For Scientific Computing
  - Numerical Analysis for Partial Differential Equations
- Thesis can be partly developed during periods of visit in foreign universities

Main references

Irregularly shaped domains and boundary conditions

- Sangalli L.M., Ramsay J.O., Ramsay T.O. (2013), “Spatial splines regression models”, *Journal of the Royal Statistical Society Ser. B, Statistical Methodology*, 75, 4, 681-703.

Incorporating a priori knowledge

- Azzimonti L., Sangalli L.M., Secchi P., Domanin M. and Nobile F. (2013), “Blood flow velocity field estimation via spatial regression with PDE penalization”, *Journal of the American Statistical Association,* 110 (511), 1057-1071.

-Azzimonti L., Nobile F., Sangalli L.M., Secchi P. (2013), “Mixed Finite Elements for spatial regression with PDE penalization”, *SIAM – ASA Journal on Uncertainty Quantification*, 2, 1, 305-335.

Manifold domains

- Ettinger B., Perotto S., Sangalli L.M. (2012), “Regression models over two-dimensional manifolds”, *Biometrika*, DOI:10.1093/biomet/asv069.

- Dassi F., Ettinger B., Perotto S., Sangalli L.M. (2013), “Mesh simplification for spatial regression of cortical surface data”, *Applied Numerical Mathematics*, Vol. 90, pp. 111-131.

- Wilhelm M., Dedè L., Sangalli L.M., Wilhelm P. (2015), “IGS: an IsoGeometric approach for Smoothing on surfaces”, *Computer Methods in Applied Mechanics and Engineering*, to appear.

Space-Time

- Bernardi M.S., Mazza G., Ramsay J.O., Sangalli L.M. (2015), “A separable model for spatial functional data with application to the analysis of the production of waste in Venice province”, Tech.rep. N. 34/2015 MOX, Dip. di Matematica, Politecnico di Milano.

Multiple realizations of the signal over manifold domains

- Lila E., Aston J.A.D., Sangalli L.M. (2015), Smooth principal component analysis over two-dimensional manifolds with an application to neuroimaging, ArXiv